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Improved Definition of
Crustal Anomalies for Magsat Data
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During the 6th quarter of this project, we reported an attempt to invert the equation for the magnetic potential in two dimensions:

$$V = \sum_{n=0}^{\infty} \left\{ \left(\frac{r}{a}\right)^{n} \left[A_n \cos (n\theta) + B_n \sin (n\theta)\right] + \left(\frac{a}{r}\right)^{n+1} \left[C_n \cos (n\theta) + D_n \sin(n\theta)\right] \right\} \tag{1}$$

$$= V_{external} + V_{internal}$$

Tests with simulated data showed that, for satellite orbits approximating that of MAGSAT, the resultant matrices were unstable; i.e., extremely small variations in the measured data led to wild excursions in the computed values for the coefficients in equation (1).

Work done during the quarter has focused on the components of the magnetic field, The expressions for the components are obtained by differentiating equation (1).

The radial component is:

$$Z = \frac{\partial V}{\partial r} = \frac{-a}{r^2} C_o + \sum_{n=1}^{\infty} \left\{ n\left(\frac{r^{n-1}}{a^n}\right) \left[A_n \cos (n\theta) + B_n \sin (n\theta)\right] \right.$$

$$\left. - (n+1)\left(\frac{a^{n+1}}{r^{n+2}}\right)\left[C_n \cos (n\theta) + D_n \sin (n\theta)\right] \right\} \tag{2}$$

and the tangential component is:

$$H = \frac{\partial V}{r\,\partial\theta} = \left\{ \sum_{n=1}^{\infty} n\left(\frac{r^{n-1}}{a^n}\right)\left[- A_n \sin(n\theta) + B_n \cos(n\theta)\right] \right.$$

$$\left. + n\left(\frac{a^{n+1}}{r^{n+2}}\right)\left[- C_n \sin(n\theta) + D_n \cos(n\theta)\right] \right\} \tag{3}$$

From equation (2) and (3), it can be seen that if Z and H are known on a circular boundary, $r = a = 1$ and the Fourier coefficients are computed for each component as:

$$Z = \sum_{n=1}^{N} \left[E_n \cos(n\theta) + F_n \sin(n\theta)\right]$$

and

$$H = \sum_{n=1}^{N} \left[G_n \cos(n\theta) + h_n \sin(n\theta)\right],$$

then the coefficients for the internal and external parts of the potential can be computed from the relations

$$E_n = nA_n - (n+1)C_n \qquad F_n = nB_n - (n+1)D_n$$

and

$$h_n = -nA_n - nC_n \qquad G_n = nB_n + nD_n \tag{4}$$

Note that we have ignored the zero order term, assuming it will be subtracted from the measurements as part of the reference field.

A somewhat similar relation is given in Chapman and Bartels for separating internal and external field effects given the vertical component and the potential.

For satellite measurements, however, the data is not obtained on a circular boundary, but rather on an orbit of varying altitude. In this case, the terms involving a (radius of the circular boundary which is arbitrary) and r (altitude of the satellite) must be included.

A solution for the coefficient of the internal and external terms in the potential can be obtained by inverting the matrix equation

$$\bar{y} = M\bar{x}$$

where

$$\bar{y} = \left[Z_1, H_1, Z_2, H_2, \ldots\ Z_m, H_m \right]^T$$

are the measured data values of H and Z

$$\bar{x} = \left[A_1, B_1, C_1, D_1, \ldots\ A_N, B_N, C_N, D_N \right]^T$$

are the coefficients for the potential and the matrix to be inverted are

$$M = \begin{bmatrix} \frac{1}{a}\cos\theta_1 & \frac{1}{a}\sin\theta_1 & -2\frac{a^2}{r^3}\cos\theta_1 - & \ldots & -(n+1)\frac{a^{n+1}}{r^{n+2}}\sin(n\theta_1) \\ -\frac{1}{a}\sin\theta_1 & \frac{1}{a}\cos\theta_1 & \frac{-a^2}{r^3}\sin\theta_1 + & \ldots & +n\frac{a^{n+1}}{r^{n+2}}\cos(n\theta_1) \\ \frac{1}{a}\cos\theta_2 & \frac{1}{a}\sin\theta_2 & -\frac{2a^2}{r^3}\cos\theta_2 - & \ldots & -(n+1)\frac{a^{n+1}}{r^{n+2}}\sin(n\theta_2) \\ \vdots & \vdots & \vdots & & \vdots \\ -\frac{1}{a}\sin\theta_m & \frac{1}{a}\cos\theta_m & -\frac{a^2}{r^3}\sin\theta_m + & \ldots & +n\frac{a^{n+1}}{r^{n+2}}\cos(n\theta_m) \end{bmatrix}$$

To test this method, a set of random numbers were used as coefficients for the internal and external parts of the potential. Using these coefficients, values of Z and H were computed along a simulated orbit path given by:

$$ALT = 68000 + 100\sin(\theta)\ \text{km}$$

512 values were computed around the circle

Two different least-squares procedures were used to solve equation (4). The first, a recursive least-squares solution was obtained by updating the equation.

$$P_{n+1} = P_n - P_n \bar{M}^T_{n+1} \left[(I + M_{n+1} P_n M^T_{n+1})\right] \bar{M}_{n+1} P_n \qquad (6)$$

$$\hat{X}_{n+1} = \hat{X}_n + P_{n+1} \bar{M}^T_{n+1} (\hat{Y}_{n+1} - \bar{M}_{n+1} \hat{X}_n)$$

In these equations P_{n+1} is the updated symmetric covariance matrix $\hat{X}_{n+1}$ is the updated coefficient vector estimate and M_{n+1} is the row vector of function values for each new observation. For the examples included here, $P_o = cI$, where I is the unity matrix and $C \approx 10^6$. Considerable errors can result if the start-up values of P_o are too small.

The second set of computations use the standard least-squares solution obtained from

$$\hat{X} = (M^T M)^{-1} M^T \hat{y}$$

In Table 1, values for the coefficients of the first 6 harmonics were assumed in generating both the internal and external terms. Both the recursive and standard least squares methods were able to recover the generating coefficients quite well, but the recursive least squares is slightly less accurate.

In Table 2, while coefficients of only the first harmonic were used in generating the external part of the field, the least-squares routines solved for coefficients of all 6 harmonics in both parts. Again the recovery was fairly accurate and the computed value for harmonics 2-6 in the external part were relatively small. It should be noted that these coefficients have the dimension of gamma-kilometers, and the actual gamma computed from their value would be very small.

The results in Table 3 show that, although 9 harmonics were used to generate the internal and external fields, a solution which looks for only 6 harmonics is not thrown off a great deal by the presence of the higher harmonics.

Finally, in Table 4, to more closely approximate the real case, the internal terms are made very large and the external

small. Here the recursive least-squares was significantly worse than the standard least-squares in accurate recovery of the external field coefficients.

During the remaining term of the contract, we will apply this technique to removal of external field variations from actual MAGSAT data.

INTERNAL		EXTERNAL	
COS	SIN	COS	SIN
-0. 2386E+06	0. 1499E+06	-0. 1985E+05	-3801.
0 1515E+06	0. 3021E+05	0. 1268E+05	-0. 3726E+05
-0 1560E+06	0. 1828E+06	0. 1557E+05	0. 7591E+05
0 1283E+06	1336.	-0. 1777E+05	-0. 5268E+05
0 5345E+05	0. 1859E+06	0. 1182E+05	0. 4543E+05
-4718.	0. 1224E+06	-0. 1019E+05	-4181.

COEFFICIENTS USED TO GENERATE DATA

INTERNAL		EXTERNAL	
COS	SIN	COS	SIN
-0. 2372E+06	0. 1493E+06	-0. 1963E+05	-4053.
0 1516E+06	0. 3020E+05	0. 1233E+05	-0. 3718E+05
-0. 1557E+06	0. 1827E+06	0. 1538E+05	0. 7581E+05
0 1284E+06	1343.	-0. 1792E+05	-0. 5266E+05
0 5355E+05	0. 1858E+06	0. 1170E+05	0. 4540E+05
-4606.	0. 1223E+06	-0. 1029E+05	-4175.

COEFFICIENTS RECOVERED USING RECURSIVE LEAST SQUARES

INTERNAL		EXTERNAL	
COS	SIN	COS	SIN
-0 2386E+06	0. 1497E+06	-0. 1965E+05	-3801.
0 1515E+06	0. 3021E+05	0. 1269E+05	-0. 3726E+05
-0 1560E+06	0. 1828E+06	0. 1557E+05	0. 7591E+05
0 1283E+06	1336.	-0. 1777E+05	-0. 5268E+05
0 5345E+05	0 1859E+06	0. 1182E+05	0. 4543E+05
-4719.	0. 1224E+06	-0. 1019E+05	-4181.

COEFFICIENTS RECOVERED USING STANDARD LEAST SQUARES

TABLE 1

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INTERNAL		EXTERNAL	
COS	SIN	COS	SIN
-0.2386E+06	0.1499E+06	-0.1985E+05	-3801.
0.1515E+06	0.3021E+05	0.0000	0.0000
-0.1560E+06	0.1828E+06	0.0000	0.0000
0.1283E+06	1336.	0.0000	0.0000
0.5345E+05	0.1859E+06	0.0000	0.0000
-4718.	0.1224E+06	0.0000	0.0000

COEFFICIENTS USED TO GENERATE DATA

INTERNAL		EXTERNAL	
COS	SIN	COS	SIN
-0.2[illegible]72E+06	0.1493E+06	-0.1962E+05	-4054.
0.1516E+06	0.3019E+05	-321.8	-12.55
-0.1557E+06	0.1827E+06	-172.7	-25.07
0.1284E+06	1340.	-156.6	-2.972
0.5355E+05	0.1858E+06	-123.7	-8.614
-4607.	0.1223E+06	-102.2	4.461

COEFFICIENTS RECOVERED USING RECURSIVE LEAST SQUARES

INTERNAL		EXTERNAL	
COS	SIN	COS	SIN
-0.2386E+06	0.1499E+06	-0.1985E+05	-3801.
0.1515E+06	0.3021E+05	0.1430	-0.1904
-0.1560E+06	0.1828E+06	0.2328E-01	-0.1910
0.1283E+06	1336.	-0.4964E-01	-0.8566E-01
0.5345E+05	0.1859E+06	-0.1638E-01	-0.5632E-01
-4719	0.1224E+06	0.4698E-01	-0.3742E-01

COEFFICIENTS RECOVERED USING STANDARD LEAST SQUARES

TABLE 2

INTERNAL		EXTERNAL	
COS	SIN	COS	SIN
-0.2386E+06	0.1499E+06	-0.1985E+05	-3801.
0.1515E+06	0.3021E+05	0.1268E+05	-0.3726E+05
-0.1560E+06	0.1828E+06	0.1557E+05	0.7591E+05
0.1283E+06	1336.	-0.1777E+05	-0.5268E+05
0.5345E+05	0.1859E+06	0.1182E+05	0.4543E+05
-4718.	0.1224E+06	-0.1019E+05	-4181.
-0.5355E+05	0.3428E+05	-3806.	-772.1
8016.	1015.	858.3	-2082.
-2178.	2094.	234.5	915.9

COEFFICIENTS USED TO GENERATE DATA

INTERNAL		EXTERNAL	
COS	SIN	COS	SIN
-0.2366E+06	0.1491E+06	-0.1973E+05	-4286.
0.1521E+06	0.3009E+05	0.1227E+05	-0.3726E+05
-0.1552E+06	0.1828E+06	0.1534E+05	0.7581E+05
0.1268E+06	1724.	-0.1798E+05	-0.5264E+05
0.5322E+05	0.1866E+06	0.1164E+05	0.4539E+05
-9769.	0.1149E+06	-0.1076E+05	-4322.

COEFFICIENTS RECOVERED USING RECURSIVE LEAST SQUARES

INTERNAL		EXTERNAL	
COS	SIN	COS	SIN
-0.2442E+06	0.1527E+06	-0.1330E+05	953.6
0.1483E+06	0.3189E+05	0.1683E+05	-0.3522E+05
-0.1562E+06	0.1841E+06	0.1871E+05	0.7710E+05
0.1267E+06	2374.	-0.1517E+05	-0.5190E+05
0.5171E+05	0.1871E+06	0.1410E+05	0.4596E+05
0.1089E+05	0.1152E+06	-8519.	-4135.

COEFFICIENTS RECOVERED USING STANDARD LEAST SQUARES

TABLE 3

ORIGINAL PAGE IS
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INTERNAL COS	INTERNAL SIN	EXTERNAL COS	EXTERNAL SIN
-0.3509E+09	0.2204E+09	-2919.	-559.0
0.2228E+09	0.4443E+08	1865.	-5479.
-0.2295E+09	0.2689E+09	2289.	0.1116E+05
0.1887E+09	0.1965E+07	-2614.	-7748.
0.7860E+08	0.2734E+09	1739.	6680.
-0.6939E+07	0.1799E+09	-1498.	-614.8

COEFFICIENTS USED TO GENERATE DATA

INTERNAL COS	INTERNAL SIN	EXTERNAL COS	EXTERNAL SIN
-0.3342E+09	0.2158E+09	0.9853E+07	-0.4277E+07
0.2388E+09	0.4744E+08	0.9269E+07	0.3360E+07
-0.2170E+09	0.2779E+09	0.6475E+07	0.8000E+07
0.1952E+09	0.1400E+08	0.1880E+07	0.9360E+07
0.7929E+08	0.2846E+09	-0.1771E+07	0.7959E+07
-0.1014E+08	0.1875E+09	-0.3246E+07	0.5666E+07

COEFFICIENTS RECOVERED USING RECURSIVE LEAST SQUARES

INTERNAL COS	INTERNAL SIN	EXTERNAL COS	EXTERNAL SIN
-0.3509E+09	0.2204E+09	-2711.	-659.7
0.2228E+09	0.4443E+08	2105.	-5765.
-0.2295E+09	0.2689E+09	2309.	0.1092E+05
0.1887E+09	0.1965E+07	-2674.	-7885.
0.7860E+08	0.2734E+09	1709.	6618.
-0.6939E+07	0.1799E+09	-1425.	-661.2

COEFFICIENTS RECOVERED USING STANDARD LEAST SQUARES

TABLE 4